A Deterministic Steady-State Error Model for the Floated Inertial Gyroscope

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The increasing accuracy of floated inertial gyros has resulted in additional sources of error becoming apparent, and the mathematical model of the gyro must be expanded to account for these effects. Two particular categories of error are discussed in detail: 1) float displacement, either lateral or rotational with respect to the case; and 2) variations in the temperature distribution. Suitable methods of parameterizing these disturbances, and some of the known physical error mechanisms which result from them, in particular those due to float relative velocities and to temperature differentials, are described. Finally, an error model suitable for use in the testing or compensation of inertial gyros is given.

Nomenclature

D_F	= error torque independent of $\bar{g}$ (dyne-cm)
D_S,D_I,D_O	= error torque per unit f_S, f_I, f_O, respectively, dyne-cm/g
D_{SS},D_{II},D_{OO}	= error torque per unit f_S^2, f_I^2, f_O^2, respectively, dyne-cm/g^2
D_{SI},D_{SO},D_{IO}	= error torque per unit f_Sf_I, f_Sf_O, f_If_O, respectively, dyne-cm/g^2
D_i	= general D coefficient
d_i	= value of D_i in reference state
e_x	= average eccentricity of float along S, cm
e_y	= average eccentricity of float along I, cm
$\bar{f}$	= specific force vector in float, g
f_x,f_y,f_z	= components of $\bar{f}$ along S, I, O, respectively, g
f_1,f_2	= shear forces on float due to fluid flow (Fig. 4)
$h(P)$	= fluid gap thickness at point P, cm
h_0	= average fluid gap thickness
I,I'	= input reference axes in housing and float, respectively
K	= gyro scale factor, nrad/dyne-cm-sec
K_{ij}	= coefficient of jth disturbance input in D_i
L	= distance between float suspension points, cm
m	= T_x'/T_x or T_y'/T_y
M	= torque on float about O' axis, dyne-cm
M_e	= error torque, dyne-cm
M_f	= total torque on float, dyne-cm
M_{tg}	= torque applied by torque generator, dyne-cm
O,O'	= output reference axes in housing and float, respectively
P	= a point on surface of float or housing
$\bar{r}$	= float displacement vector
rst	= gyro reference state
R	= radius of float, cm
st	= gyro state
S,S'	= spin reference axes on housing and float, respectively
$T(P)$	= temperature disturbance at P, °C
T_A	= average $T(P)$ in S-I plane, °C
$T_g(P)$	= temperature at surface of gyro, °C
$T_0(P)$	= reference temperature at surface of gyro, °C
T_{Sz},T_{Tz}	= $T(P)$ at $-O$ and $+O$ ends of housing, respectively, °C
T_x	= disturbance temperature differential across housing along S axis, °C
T_x'	= disturbance temperature differential across float along S axis, °C
T_y	= disturbance temperature differential across housing along I axis, °C
T_y'	= disturbance temperature differential across float along I axis, °C
W_w	= wheel motor power, w
W_{tg}	= torque generator power, w
x,y,z	= components of $\bar{r}$ along S, I, and O, respectively, cm
α_x,α_y	= reference-state misalignment of float about S and I axes, respectively, rad
β	= coefficient of cubical expansion of fluid, °C^{-1}
$\theta_x,\theta_y,\theta_z$	= displacement of float about S, I, and O axes, respectively, rad
μ_0	= fluid viscosity in reference state, poise
ρ_0	= fluid density in reference state, gm/cm^3
$\omega_S,\omega_I,\omega_O$	= inertially referred rates about S, I, and O axes, respectively, nrad/sec (1 nrad/sec $\simeq$ 0.002°/hr $\simeq$ 0.014 meru)

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Introduction

When a gyro is under test, it is assumed that it can be adequately represented by an equation of fixed functional form relating the input and output variables. This model, to be of practical use, must contain a manageably small number of coefficients, which can be measured in a suitable calibration test. The model and the measured coefficients then should form a representation of the individual gyro usable for predicting its behavior in the intended application. As the accuracy of gyros is improved, it becomes necessary to amplify the mathematical model so as to include a number of additional coefficients, representing the sensitivity of a gyro to various environmental variables. It is these additional sensitivities with which this paper is concerned, in respect to the single-degree-of-freedom floated rate-integrating gyroscope of conventional cylindrical design.[1]

The complete representation of the gyro is prohibitively complex. This paper deals only with certain steady-state errors; transient and kinematic errors are excluded. The primary disturbance signals are the components of specific force $\bar{f}$ acting on the float. Two other sets of variables that cause error are also considered. One is the position of the float with respect to the housing, and its rate of change. While such a misalignment or relative motion must be caused in the first place by a transient input, the high damping coefficients are such that these effects linger long after the cause has vanished, and they are considered as quasi-steady-state effects. The second disturbing input to be considered is gyro temperature distribution, and in this case the first task is to find a simple yet adequate method of specifying and measuring it.

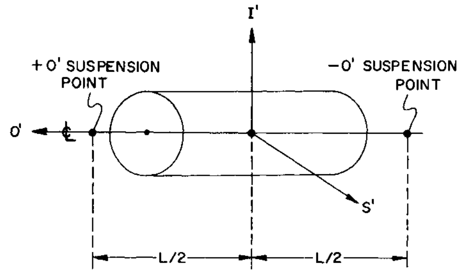

Fig. 1 Float axis system.

This paper stresses the concept of a reference state, a condition of the gyro in which all variables are held to some fixed standard value. The effects of variation of individual variables can be separately calibrated using the reference state as datum.

Representation of Gyro Disturbance Input Signals

The gyro fundamental coordinate system (Fig. 1) is a set of axes I', O', and S' fixed in the float. The spin axis S' is defined in direction (but not, for this purpose, in position) by the axis of rotation of the wheel. The output axis O' is defined in both direction and position as the center-line of the cylindrical outer surface of the float. It is about this axis that the float will rotate most freely when subjected to a torque. The output axis is orthogonal to the spin axis within the constructional accuracy of the float. The input axis I' is orthogonal to S' and O'. The origin of coordinates for the float is taken as that point on the output axis exactly halfway between the planes of suspension, at which points the float relative position may be measured.

A similar set of axes is imbedded in the housing, and designated I, O, and S. These, the reference axes for practical purposes, are defined as follows. Let the float be aligned so that all electromagnetic float-position signals are at null. The axes I, O, and S are then congruent with axes I', O', and S'. This must be done, of course, under some predefined standard electrical conditions. The reference state of the gyro with respect to these variables is then defined as that in which the float is at rest with respect to the housing, and the two sets of axes are congruent.

Owing to imperfections in construction, the alignment procedure defined above does not guarantee that the cylindrical outer wall of the float will be either concentric with or parallel to the cylindrical inner wall of the housing. Thus, the fluid-gap thickness in the reference state must be treated as a function of position $h(P)$; $h(P)$ is not easily measured directly, but it can be inferred by use of the float-motion mechanisms described later.

The specific force $\bar{f}$ acting at the float may be resolved along the S, I, and O axes into the components f_x, f_y, and f_z, respectively. The location of the float with respect to the housing can be measured as a displacement vector $\bar{r}$ (Fig. 2) with coordinates x, y, z and a rotation which, being small, can be treated as a vector with components θ_x, θ_y, θ_z.

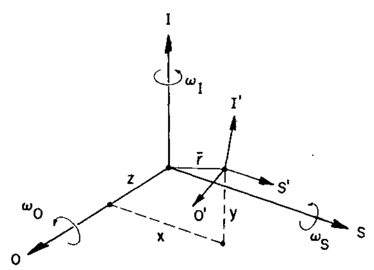

Fig. 2 Float displacement.

The representation of gyro temperature distribution is particularly complex. For convenience, the discussion is restricted to conventional cylindrical gyros, symmetrically constructed with respect to the output axis, and symmetrically mounted by central bosses at each end. Symmetrical construction of a gyro makes it more resistant to both thermal and mechanical disturbances, so that precision gyros are generally built this way. Internal heat-flux disturbance sources include, for example, wheel power; external sources include convection losses, radiation, and dissipation in contiguous components. The relationship of heat flux to temperature is analogous to that between current and voltage in a network, but its analysis is considerably more complex.[2] In the following, the gyro surface temperature $T_g(P)$, a function of the surface coordinate point P, is assumed to be measurable. This, together with a knowledge of the internal power consumption is sufficient to specify the entire temperature state of the gyro.

It is necessary to define a reference temperature state $T_0(P)$ as a datum distribution, and we prefer to choose one that can easily be established under test conditions. Provided it can be repeatably established, it is not necessary to know $T_0(P)$ completely. The main concern is with

$$T(P) = T_g(P) - T_0(P) \tag{1}$$

where $T(P)$ is a temperature disturbance which can be set up by such things as changed convection patterns, radiant heat, or many other causes of variation in state. For a well-insulated, symmetrically constructed gyro, $T(P)$ can be adequately observed using a set of six temperature measurements $T(P_1) \ldots T(P_6)$ as indicated in Fig. 3, from which the following more physically relevant temperature-state parameters can be derived:

$$T_A = \tfrac{1}{4}[T(P_1) + T(P_2) + T(P_3) + T(P_4)]$$

$$T_x = T(P_2) - T(P_4) \qquad T_y = T(P_1) - T(P_3)$$

$$T_{Sz} = T(P_5) \qquad T_{Tz} = T(P_6)$$

Other significant sources of disturbance such as the wheel power W_w, and torquer power W_{tg}, are more easily handled, and will not be further discussed. This is not intended to be an exclusive list; other sources of possible error include electric, magnetic, and pressure fields.

Mathematical Representation of the Gyro

In the torque-to-null mode of operation, the steady-state law of the gyro is basically

$$\omega_I = KM_f \tag{2}$$

In practice, a number of error torques must also be taken into account; they form the subject of this paper. Thus,

$$M_f = M_{tg} + M_e \tag{3}$$

where KM_{tg} is the indicated output rate and M_e represents error torque.

The torque M_e is the resultant of many independent effects; it can be functionally expressed as follows:

$$M_e = \phi(f_x,f_y,f_z,x,y,z,\theta_x,\theta_y,\theta_z,\dot{x},\dot{y},\dot{z},\dot{\theta}_x,\dot{\theta}_y,\dot{\theta}_z, T_A,T_x,T_y,T_{Sz},T_{Tz},W_w,W_{tg} \ldots)\dagger \tag{4}$$

M_e represents an error torque physically present on the float. In addition, error can arise because of incorrect or time-varying calibration of the torque-generator output M_{tg}. This type of error is outside the scope of this paper.

All the disturbance sources included in Eq. (4) can be expected to vary over only a small range, with the exception

† Errors have also been observed proportional to the derivatives of T_x and T_y; these are omitted as being transient in nature.

of the components of $\bar{f}$ which in some applications vary quite widely. It is therefore logical, and also customary, to regard $\bar{f}$ as the primary source of disturbance. M_e can (for a good instrument) be expanded as a Maclaurin's series in f_x, f_y, and f_z, about $\bar{f} = 0$, viz:

$$M_e = M_e\Big|_{\bar{f}=0} + \frac{\partial\phi}{\partial f_x}\Big|_{\bar{f}=0} f_x + \frac{\partial\phi}{\partial f_y}\Big|_{\bar{f}=0} f_y + \ldots + \frac{1}{2}\frac{\partial^2\phi}{\partial f_x^2}\Big|_{\bar{f}=0} f_x^2 + \frac{\partial^2\phi}{\partial f_x \partial f_y}\Big|_{\bar{f}=0} f_x f_y + \ldots + \frac{1}{6}\frac{\partial^3\phi}{\partial f_x^3}\Big|_{\bar{f}=0} f_x^3 + \frac{1}{3}\frac{\partial^3\phi}{\partial f_x^2 \partial f_y}\Big|_{\bar{f}=0} f_x^2 f_y + \frac{\partial^3\phi}{\partial f_x \partial f_y \partial f_z}\Big|_{\bar{f}=0} f_x f_y f_z + \ldots \quad (5)$$

The relative significance of the terms in the expansion depends on the nature of the function in Eq. (4); experience and physical reasoning indicate that all terms up through second order may be present, and at high accelerations we can expect that nonlinearities in the internal physical mechanisms of the gyro will create significant higher order terms. The currently accepted model includes terms through the second order; in more usual notation the model is

$$M_e = D_F + D_S f_S + D_I f_I + D_O f_O + D_{SS} f_S^2 + D_{II} f_I^2 + D_{OO} f_O^2 + D_{SI} f_S f_I + D_{SO} f_S f_O + D_{IO} f_I f_O \quad (6)$$

The second-order terms are attributable largely to a lack of rigidity, symmetry, and homogeneity in the construction of the float, creating elastic anisotropy and nonlinearity. It is believed that the second-order coefficients are quite stable, and relatively insensitive to the various disturbance sources. The model to be studied here thus consists of the terms through first order. Some of these coefficients are sensitive to a number of disturbances, including those listed in Eq. (4); the general coefficient D_i can be written

$$D_i = \phi_i(x,y,z,\theta_x,\theta_y,\theta_z,\dot{x},\dot{y},\dot{z},\dot{\theta}_x,\dot{\theta}_y,\dot{\theta}_z,T_A,T_x,T_y \ldots W_w \ldots) \quad (7)$$

Since each of these disturbances should vary over a very limited range in a precision gyro, it will be satisfactory for the most part to represent D_i by the first-order terms of the Taylor series, expanded about the reference state of the gyro as defined (and denoted as *rst*):

$$D_i = d_i + \frac{\partial\phi_i}{\partial x}\Big|_{rst} x + \frac{\partial\phi_i}{\partial y}\Big|_{rst} y + \ldots + \frac{\partial\phi_i}{\partial\theta_x}\Big|_{rst} \theta_x + \ldots \quad (8)$$

or

$$D_i = d_i + K_{i1}x + K_{i2}y + K_{i3}z + K_{i4}\theta_x + K_{i5}\theta_y + K_{i6}\theta_z + K_{i7}\dot{x} + K_{i8}\dot{y} + K_{i9}\dot{z} + K_{i10}\dot{\theta}_x + K_{i11}\dot{\theta}_y + K_{i12}\dot{\theta}_z + K_{i13}T_A + K_{i14}T_x + K_{i15}T_y + K_{i16}T_{Sz} + K_{i17}T_{Tz} + K_{i18}W_w + K_{i19}W_{tg} \quad (9)$$

where $i \rightarrow F, I, O$, or S.

The constants d_F, d_I, d_O, and d_S correspond to the conventional drift coefficients, with the proviso that they are measured with the gyro in the reference state. The K_{ij} terms of Eq. (9) represent the influences of the various disturbing inputs on each coefficient. In the following sections, some of the physical effects that are known to occur are described.

Physical Error Mechanisms

Errors as a Function of Float Displacement

The principal forces acting on the float are gravitational, electromagnetic, mechanical contact, and fluid pressure and shear forces. The gravitational field is assumed constant in all these discussions, having already been accounted for in Eq. (5). Viscous forces occur only when relative motion exists between float and fluid. If the float is displaced laterally by a small fixed amount (x, y, z) from its standard position, and is at rest there, the main sources of force and torque acting on the gyro are then the mechanical-contact forces due to flex leads, bias adjustment devices, etc., the fluid pressure, and the electromagnetic forces and torques set up in the suspension, torque-generator and signal-generator mechanisms. There will inevitably be some change in net torque applied to the float as a result of lateral displacement, due to small changes in these forces. Elastic forces of this nature exist independent of $\bar{f}$; hence the terms involved are K_{F1}, K_{F2}, and K_{F3}. The change in shape of the flex leads due to lateral offset, of the float may also cause a change in the torque applied to the float as a result of the specific force at the flex leads. This would show up in the K_{I1}, K_{I2}, K_{I3}; K_{O1}, K_{O2}, K_{O3}; K_{S1}, K_{S2}, K_{S3} terms.

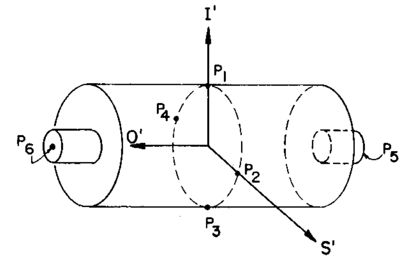

Fig. 3 Gyro temperature - measuring points.

It is important to emphasize that the theory is linked to small displacements, i.e., a small percentage of gap width. If large displacements occur, then the simple first-order expansion of Eq. (9) will not hold.

Rotation of the float about any axis will cause a change in the state of the flex leads and the magnetic circuits, and the above remarks apply equally here, in regard to K_{F4}, K_{F5}, and K_{F6}, and the corresponding terms in D_I, D_O, and D_S. (In addition the misalignment angles of the float cause the float axes to become skewed with respect to the case reference axes and, since input rates are applied with reference to the case axes, the float input axis will now pick up some of the input rates ω_O and ω_S about the case O and S axes, respectively. Kinematic errors are outside the scope of this paper.)

Error as a Result of Float Lateral Velocity ($\dot{x}$, $\dot{y}$, or $\dot{z}$)

For some time it has been known that an error torque is created due to the existence of $\dot{x}$ or $\dot{y}$. Recent work has thrown additional light on the effect.

First discussed is the effect of $\dot{x}$. Consider the cross section of the gyro in Fig. 4. In the reference state, the center of the float is eccentric by an amount e_y in the I direction with respect to the center of the housing cavity. (The e_y will in general be a function of z; this fact does not affect the present discussion if an average value is used.) The fluid gap is greatly exaggerated in relative size in Fig. 4 to facilitate the illustration. Typically, the gap thickness h_0 is less than 1% of float diameter.

The effect of $\dot{x}$ is to increase the pressure in the fluid at point A, and decrease it at B, causing the fluid to flow in the

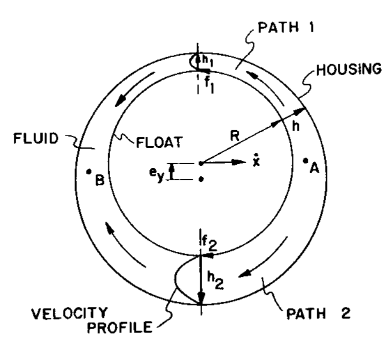

Fig. 4 Effect of float lateral velocity.

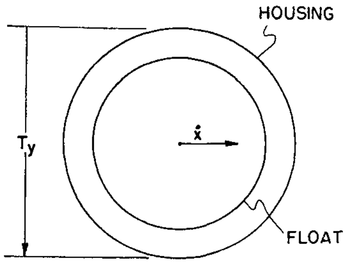

Fig. 5 Motion with temperature gradient.

directions shown via paths 1 and 2 from A to B. These fluid velocities create drag forces f_1 and f_2 on the float. The average velocity at each side is approximately proportional to h^2 at that side; consequently, the velocity gradient at the float wall is proportional to h, which means that $f_2 > f_1$ in an amount proportional to $h_2 - h_1 = 2e_y$. Consequently, under the conditions of Fig. 4 there exists a clockwise torque proportional to $e_y\dot{x}$. In the case of infinitely long cylinders, this torque per unit length of cylinder‡ is

$$M = (6\pi R^3\mu_0/h_0^3)e_y\dot{x} \text{ dyne-cm} \tag{10}$$

On an actual float, flow round the ends will reduce this. Since the effect clearly exists independent of $\bar{f}$, it forms part of the coefficient K_{F7} [Eq. (9)]. A corresponding effect for $\dot{y}$ in the presence of an eccentricity e_x forms part of the coefficient K_{F8}.

In the above effects, it must be emphasized that the velocities are extremely small. Eccentricities, on the other hand, may be quite high, possibly up to the order of 0.002 cm. Hence the velocity can exist for some time before appreciably altering the eccentricity.

Typical values for Eq. (10) are $h_0 = 0.015$ cm; $R = 3$ cm; $\mu = 40$ poise; $e_y = 0.001$ cm; $L = 6$ cm, for which $M = 2 \times 10^7\dot{x}$ dyne-cm. If $\dot{x} = 0.01\ \mu$ in./sec $= 2.5 \times 10^{-8}$ cm/sec, $M = 0.5$ dyne-cm, a significant term in most gyros.

In the above treatment, e_y is the eccentricity existing with the gyro in the reference state. If in addition there is a displacement y, the total torque will be proportional to

$$(e_y + y)\dot{x} = e_y\dot{x} + y\dot{x} \tag{11}$$

The first term is identified with $K_{F7}\dot{x}$, but the second term is a product not included in Eq. (9). However, the e_y term will usually be dominant. A similar term exists in $x\dot{y}$, and exactly the same comments apply.

It has been shown experimentally that even if the float is concentric, but there exists a temperature differential T_y on the housing (Fig. 5), $\dot{x}$ will generate a significant torque. This effect is a bilinear term, not included in Eq. (9); it depends on the product $T_y\dot{x}$. A similar effect is observed in the term $T_x\dot{y}$.

Experience indicates that $\dot{z}$, motion along the output axis, also generates an error torque. It is presumed that asymmetries on the float give rise to a net fluid torque about the O axis. Like the $\dot{x}$ and $\dot{y}$ terms, it can exist independent of $\bar{f}$.

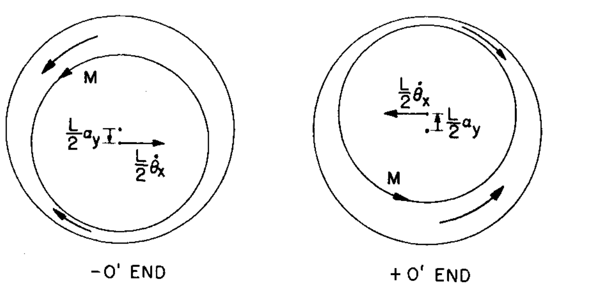

Fig. 6 Generation of torque due to angular rotation. (Both sections viewed from same direction.)

‡ A result due to R. Manasse of MIT in 1951.

Errors as a Result of Float Angular Velocity ($\dot{\theta}_x$, $\dot{\theta}_y$, or $\dot{\theta}_z$)

The torque applied to the float under rotation $\dot{\theta}_z$ is a well known basic property of the gyro. Hence K_{F12} can be identified with the rotational damping coefficient about O. Experiment indicates that rotation $\dot{\theta}_x$ can also generate large torques, under certain circumstances. The mechanism responsible is the same as the one described for the error due to $\dot{x}$. The effect arises when the float rotates about the x axis, while it is also misaligned by an angle α_y with respect to the housing inner cylindrical wall. (As noted before, the float will not in general be aligned with the housing in the reference condition.) The situation is depicted in Fig. 6 where the fluid flow pattern analogous to Fig. 4 is shown for each end of the float.

If the two planes of observation are separated by a distance L, then the eccentricity at the $-O'$ suspension point is $-\frac{1}{2}L\alpha_y$, while the eccentricity at the other end will be of opposite sense, i.e., $\frac{1}{2}L\alpha_y$. Under rotation $\dot{\theta}_x$, the $-O'$ end of the float will acquire a velocity $\frac{1}{2}L\dot{\theta}_x$, whereas the $+O'$ end will acquire a velocity $-\frac{1}{2}L\dot{\theta}_x$. The figure illustrates (using Fig. 4 as a reference) that both ends are subject to an anticlockwise torque (i.e., a negative torque), which means that the float as a whole is subject to an anticlockwise torque, proportional to $\alpha_y\dot{\theta}_x$. An exactly similar process may generate a torque due to $\dot{\theta}_y$.

The angle α_y is assumed to exist at the reference state. If there is an error θ_y also present, then the torque becomes proportional to

$$(\alpha_y + \theta_y)\dot{\theta}_x = \alpha_y\dot{\theta}_x + \theta_y\dot{\theta}_x \tag{12}$$

The first term corresponds to $K_{F10}\dot{\theta}_x$, Eq. (9), but the second term is a product term not included in Eq. (9). However, typically, θ_y should be much smaller than α_y.

Errors as a Result of Change in Central Temperature T_A

The error terms $K_{i13}T_A$, Eq. (9), represent the effect on the coefficients of a steady-state change in central temperature T_A, while all other disturbances remain unchanged.

An input T_A raises the central section through a temperature increment. The principal effect is to change physical properties; viscosity and density are seriously affected, and it is possible that magnetic and elastic properties are also significantly changed. These changes, acting on the small asymmetries and inhomogeneities of the float, may significantly affect both the restraint and mass unbalance torques (i.e., coefficients K_{F13}, K_{I13}, K_{S13}).

Errors as a Result of a Change in Temperature Differential Across the Gyro (T_x or T_y)

T_x can be caused by the application of a source of heat (or heat loss) to one side of the gyro only, along the spin axis. It consists in one side of the gyro rising in temperature while the other side suffers a corresponding decrease. It principally affects D_I. The mechanism of this error has been studied extensively.[3]

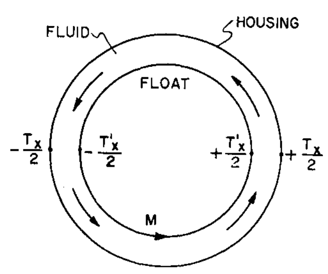

Fig. 7 Fluid convection torque.

Two effects occur. The first and usually larger effect is the thermal strain of the float due to the decrease of density on the hotter side and the increase of density on the cooler side. It can be shown[3] that the float is then subject to a torque, under the influence of specific force component f_y, whose sense is such that the colder side tends to move in the direction of f_y.

The second torque results from the convection current set up in the fluid gap (Fig. 7). It can be shown§ for the case of parallel infinite cylinders, and assuming a sinusoidal temperature distribution, that this torque is of the form

$$M = (\pi/12)T_x(1 + 2m)R^2\rho_0\beta h_0 g \text{ dyne-cm per unit length} \tag{13}$$

The second torque acts in the same sense as the first. The two effects together account for the coefficient K_{I14}. An exactly similar effect along the quadrature axis under a temperature differential T_y accounts for the coefficient K_{S15}.

Other Errors

Errors can occur as a result of a change in the temperature of mounting surfaces T_{Sz} and T_{Tz}. In a symmetrical gyro, the effects are likely to be analogous to those of T_A, but since the two ends of a gyro are dissimilar, the effects may be quite different. There are also errors as a result of internal power-dissipation variations W_w, W_{tg}, etc. The definition requires that these sensitivities be measured while other disturbances (e.g., float motion, T_A, T_x, etc.) remain unaffected. These dissipations significantly affect the coefficients D_F, D_I, and D_S. The mechanisms are no doubt similar to those relating to T_A, T_{Sz}, and T_{Tz}.

Conclusions

In order to allow for the possibility that environmental or other factors may affect gyro performance, the model must be expanded to include these variables. The model below includes those coefficients which are known to exist, or are considered likely to exist in some instances.

$$\omega_I = KM_{tg} - \theta_x\omega_O + \theta_z\omega_S + K[D_F(st) + D_I(st)f_I + D_O(st)f_O + D_S(st)f_S + \text{higher terms}] \tag{14}$$

§ A result due to D. W. Seaton, Vitro Labs, in 1965.

where st is a variable representing the gyro state. The four state-dependent coefficients are expanded as follows:

$$D_F(st) = d_F + K_{F1}x + K_{F2}y + K_{F3}z + K_{F4}\theta_x + K_{F5}\theta_y + K_{F6}\theta_z + K_{F7}\dot{x} + K_{F8}\dot{y} + K_{F9}\dot{z} + K_{F10}\dot{\theta}_x + K_{F11}\dot{\theta}_y + K_{F12}\dot{\theta}_z + K_{F13}T_A + K_{F16}T_{Sz} + K_{F17}\, T_{Tz} + K_{F18}W_w + K_{F19}W_{tg} + K_{F1\cdot8}x\dot{y} + K_{F2\cdot7}y\dot{x} + K_{F7\cdot15}\dot{x}T_y + K_{F8\cdot14}\dot{y}T_x + K_{F4\cdot11}\theta_x\dot{\theta}_y + K_{F5\cdot10}\theta_y\dot{\theta}_x \tag{15}$$

$$D_I(st) = d_I + K_{I13}T_A + K_{I14}T_x + K_{I16}T_{Sz} + K_{I17}T_{Tz} + K_{I18}W_w \tag{16}$$

$$D_O(st) = d_O + K_{O14}T_x(?) + K_{O15}T_y(?) + K_{O16}T_{Sz} + K_{O17}T_{Tz} + K_{O18}W_w \tag{17}$$

$$D_S(st) = d_S + K_{S13}T_A + K_{S15}T_y + K_{S16}T_{Sz} + K_{S17}T_z + K_{S18}W_w \tag{18}$$

The presence of a sensitivity to a given disturbance input in Eqs. (15–18) may or may not be important depending on a) whether the environment of the gyro in its final application will ever subject the gyro to any significant amount of that input, and b) whether the test environment may ever subject the gyro to such a disturbance. If the system environment contains the disturbance, it will usually be desirable to calibrate the gyro against that particular input; if the system does not contain it, but the test environment does, then the best solution is to devise an improved test procedure. If neither the system nor the test environment contain such a disturbance, it can be safely ignored (although it should never be forgotten).

References

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